Universität Bielefeld

Spectral and Transport properties from Lattice QCD

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Spectral and transport properties in the QGP

Thermal dilepton rate

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2(e^{\omega/T}-1)} \rho_{\mathbf{V}}(\omega, \mathbf{T})$$

Thermal photon rate

$$\omega \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x\mathrm{d}^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T}-1} \rho_V(\omega = |\vec{k}|, T)$$

Transport coefficients are encoded in the same spectral function

→ Kubo formulae

Diffusion coefficients:

$$DT = \frac{T}{2\chi_q} \lim_{\omega \to 0} \frac{\rho_{\mathrm{ii}}(\omega)}{\omega}$$

Need to determine vector-meson spectral functions

On the lattice only correlation functions can be calculated

→ spectral reconstruction required

Vector-meson spectral function – hard to separate different scales

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \rho(\omega,\vec{p},T) K(\tau,\omega,T)$$

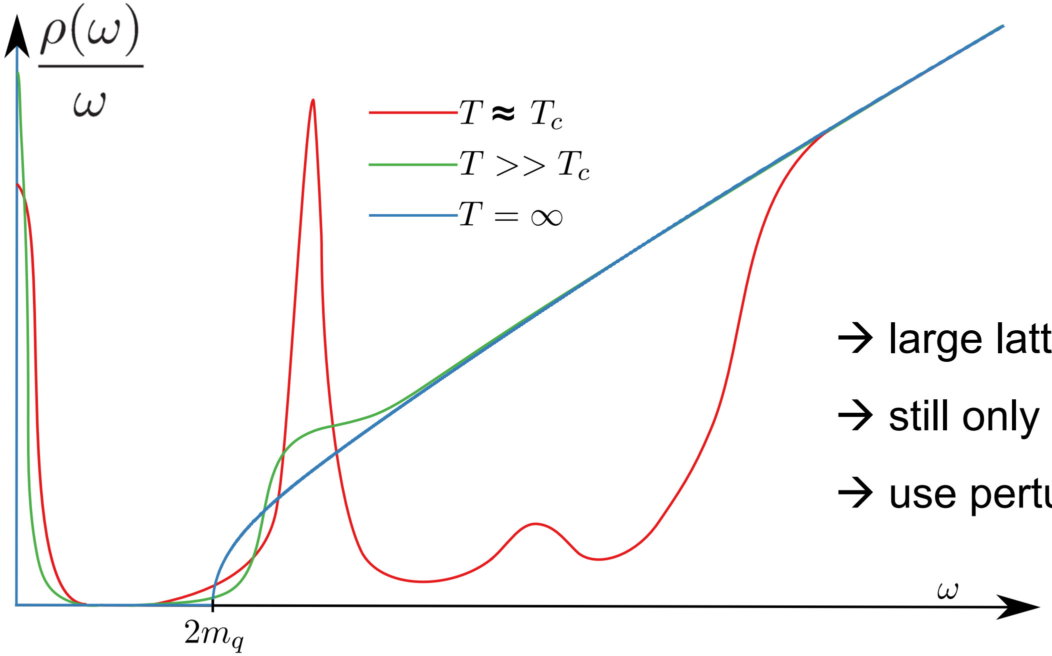
$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Different contributions and scales enter in the spectral function

- **continuum at large frequencies**
- **possible bound states at intermediate frequencies**
- **transport contributions at small frequencies**
- **in addition cut-off effects on the lattice**

notoriously difficult to extract from correlation functions

Spectral functions in the QGP



$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0})\rangle$$
$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\Gamma_\mu \psi(\tau,\vec{x})$$

→ large lattices and continuum extrapolation needed

→ still only possible in the quenched approximation

→ use perturbation theory to constrain the UV behavior

(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

H-T.Ding, F.Meyer, OK, PRD94(2016)034504
H.T.Ding, A.Francis, OK et al., PRD83(2011)034504

Quenched SU(3) gauge configurations at (separated by 500 updates)

three temperatures in the QGP: T/T_c = 1.1, 1.3 and 1.5

lattice size $N_\sigma^3 N_\tau$ with $N_\sigma = 32 - 192$
$N_\tau = 16 - 64$

Temperature: $T = \frac{1}{aN_\tau}$

non-perturbatively O(a) clover improved Wilson fermions

non-perturbative renormalization constants

quark masses close to the chiral limit $\kappa \simeq \kappa_c \Leftrightarrow m_{\overline{MS}}/T[\mu=2\text{GeV}] \approx 0.1$

fixed aspect ratio $N_\sigma/N_\tau = 3$ and 3.43 to allow continuum limit at finite momentum:

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

constant physical volume $(1.9\text{fm})^3$

Vector correlation function on large & fine lattices

H-T.Ding, F.Meyer, OK, PRD94(2016)034504
H.T.Ding, A.Francis, OK et al., PRD83(2011)034504

PRACE-Project:
Thermal Dilepton Rates and
Electrical Conductivity in the QGP

(JUGENE Bluegene/P in Jülich) + BG/Q in Jülich + Bielefeld GPU-Cluster + …

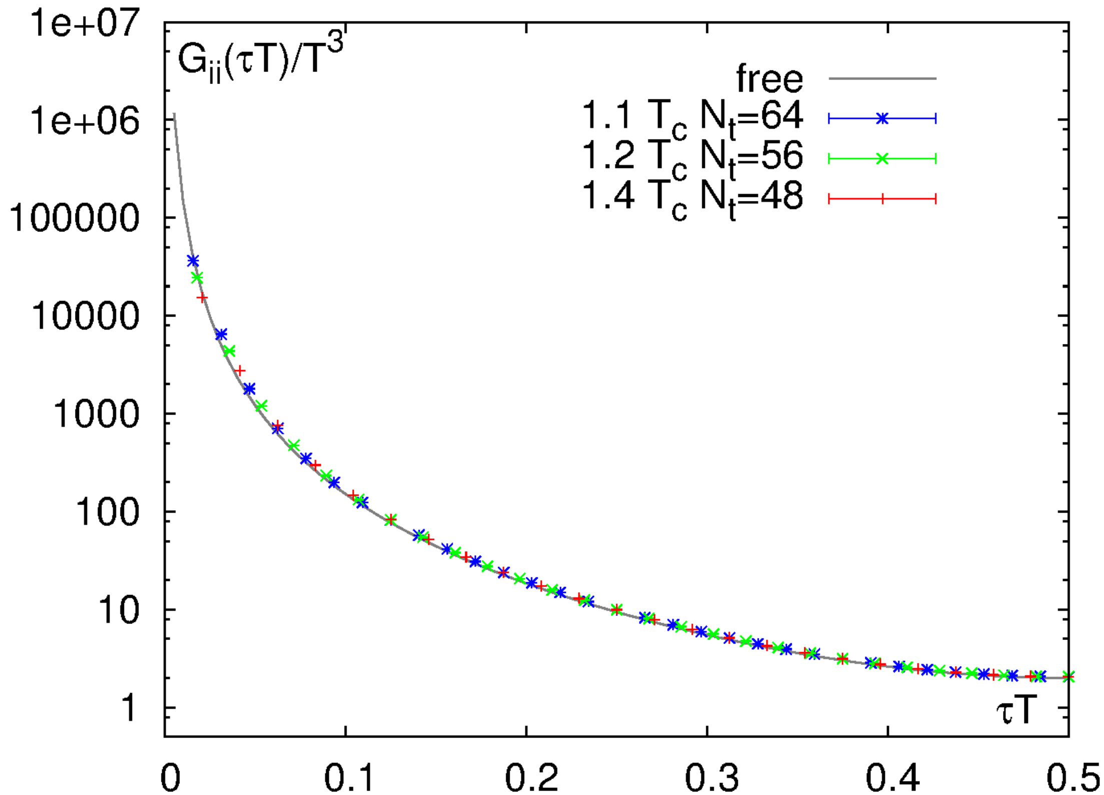
	N_τ	N_σ	β	κ	$T\sqrt{t_0}$	$T/T_c\vert_{t_0}$	Tr_0	$T/T_c\vert_{r_0}$	confs
1.1 T_c	32	96	7.192	0.13440	0.2796	1.12	0.8164	1.09	314
	48	144	7.544	0.13383	0.2843	1.14	0.8169	1.10	358
	64	192	7.793	0.13345	0.2862	1.15	0.8127	1.09	242
1.3 T_c	28	96	7.192	0.13440	0.3195	1.28	0.9330	1.25	232
	42	144	7.544	0.13383	0.3249	1.31	0.9336	1.25	417
	56	192	7.793	0.13345	0.3271	1.31	0.9288	1.25	273
1.5 T_c	24	128	7.192	0.13440	0.3728	1.50	1.0886	1.46	340
	32	128	7.457	0.13390	0.3846	1.55	1.1093	1.49	255
	48	128	7.793	0.13340	0.3817	1.53	1.0836	1.45	456

Scale setting using r_0 and t_0 [A.Francis, M.Laine, T.Neuhaus, H.Ohno PRD92(2015)116003]

fixed aspect ratio N_σ/N_τ = 3 and 3.43 to allow continuum limit at finite momentum:

$$\frac{\vec{p}}{T} = 2\pi \vec{k}\frac{N_\tau}{N_\sigma}$$

constant physical volume $(1.9\text{fm})^3$

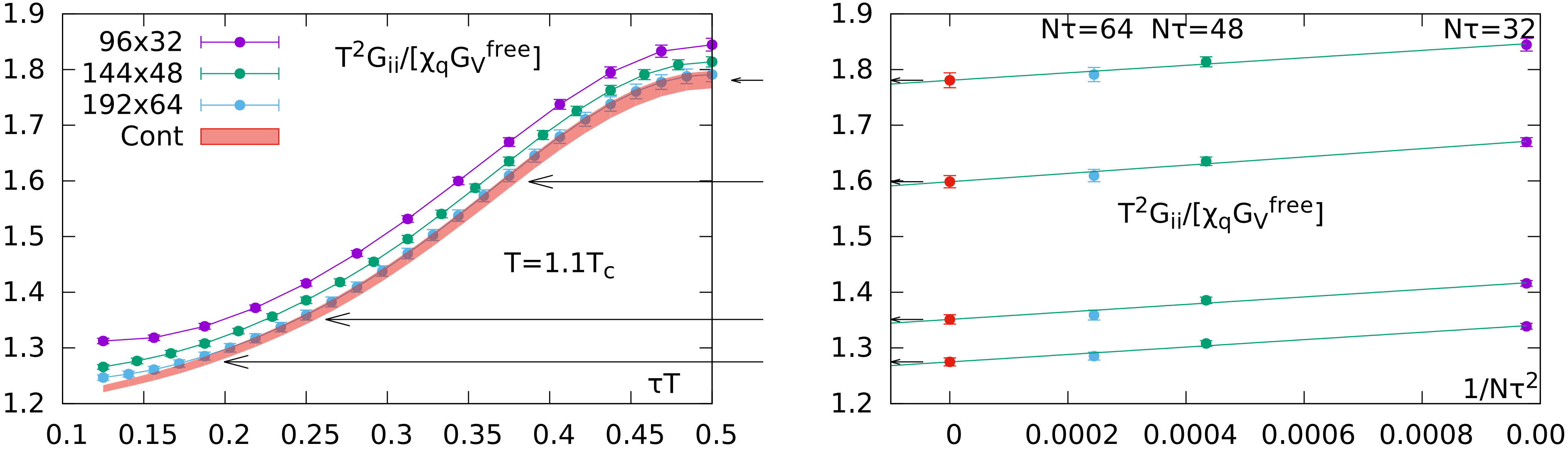


compared to free (non-interacting) correlator:

$$G_V^{free}(\tau) = 6T^2\left(\pi(1-2\tau T)\frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)}\right)$$

hard to distinguish differences due to different orders of magnitude in the correlator

→ in the following we will use $G_V^{free}(\tau)$ as a normalization



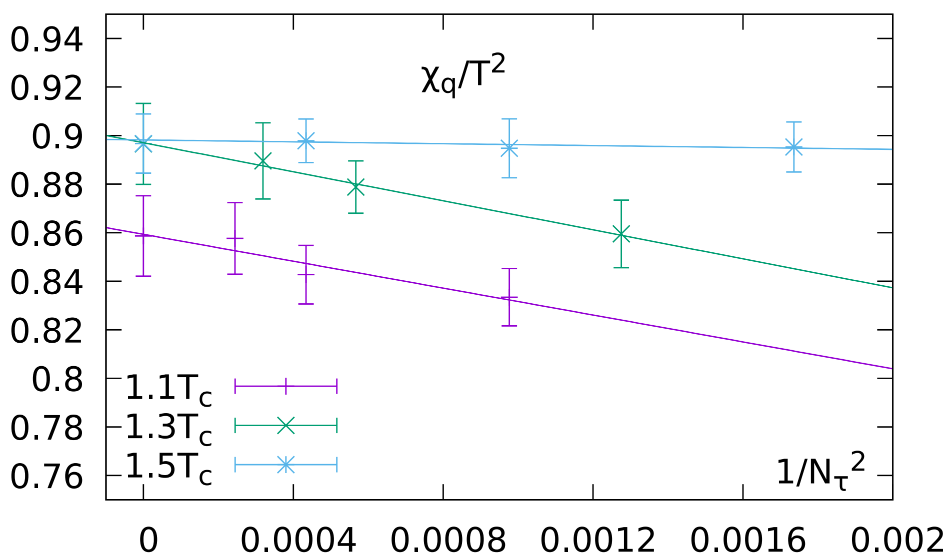
correlators normalized by quark number susceptibility χ_q independent of renormalization

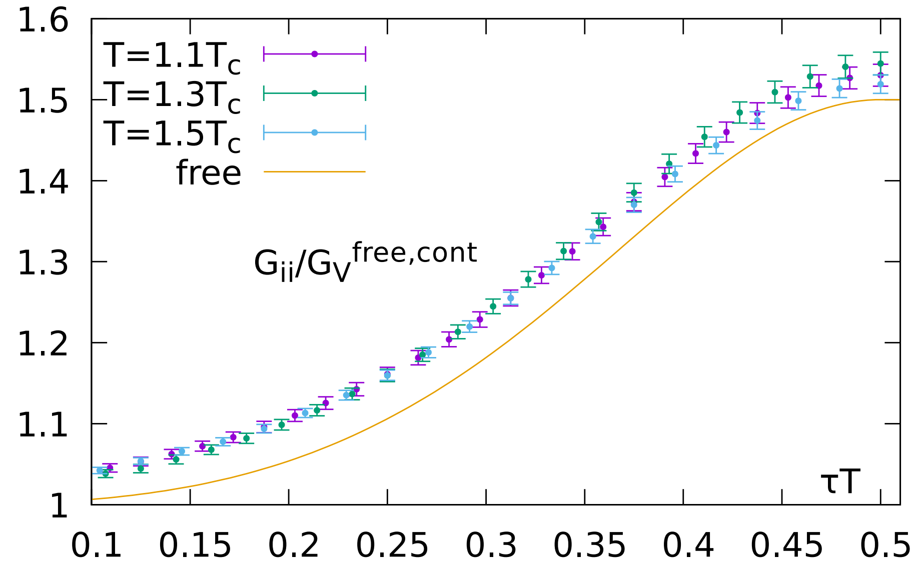
and by the free non-interacting correlator $G_V^{free}(\tau)$

we interpolate the correlator for each lattice spacing

and perform the continuum limit a→0 at each distance τT

cut-off effects are visible at all distances on finite lattices continuum extrapolated results available for three temperatures in the QGP





similar behavior in this temperature region

main difference due to different quark number susceptibility χ_q/T^2

→ indications for a weak T-dependence of the temperature scaled

electrical conductivity and thermal dilepton rates

Improve the UV behavior of the spectral function using perturbation theory:

At very high energies, due to asymptotic freedom

→ perturbation should be working

→ thermal effects should be suppressed

→ "vacuum physics"

5-loop vacuum spectral function:

$$\rho_V(\omega) = \frac{3\omega^2}{4\pi} R(\omega^2)$$

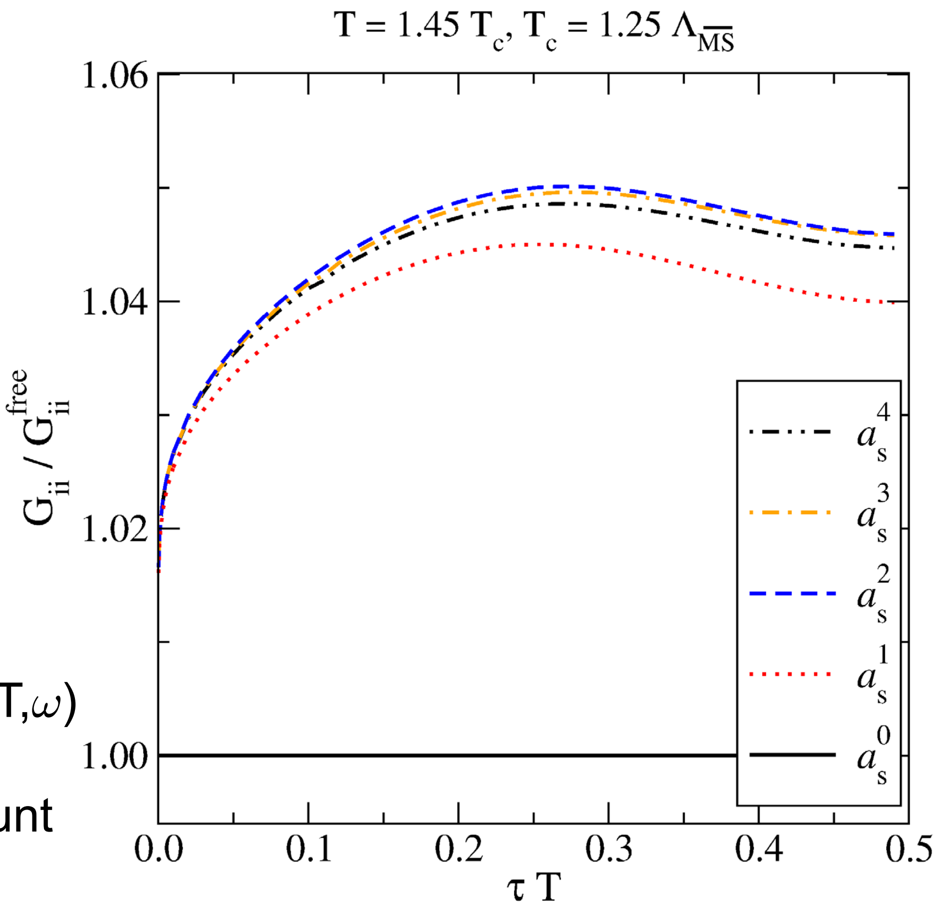
$$\begin{aligned} R(\omega^2) &= r_{0,0} + r_{1,0}\, a_s + (r_{2,0} + r_{2,1}\,\ell)\, a_s^2 \\ &+ (r_{3,0} + r_{3,1}\,\ell + r_{3,2}\,\ell^2)\, a_s^3 \\ &+ (r_{4,0} + r_{4,1}\,\ell + r_{4,2}\,\ell^2 + r_{4,3}\,\ell^3)\, a_s^4 + \mathcal{O}(a_s^5) \end{aligned}$$

using 3-loop α_s and l=log(μ^2/ω^2)

using a renormalization scale μ=(1..5)max(πT,ω)

taking leading order thermal effect into account

$$\rho_{ii}^{(T)}(\omega) \equiv \frac{3\omega^2}{4\pi}[1 - 2n_{\mathrm{F}}(\frac{\omega}{2})]R(\omega^2) + \pi\chi_{\mathrm{q}}^{\mathrm{free}}\omega\delta(\omega)$$



[Y.Burnier and M.Laine, arXiv 1201.1994]

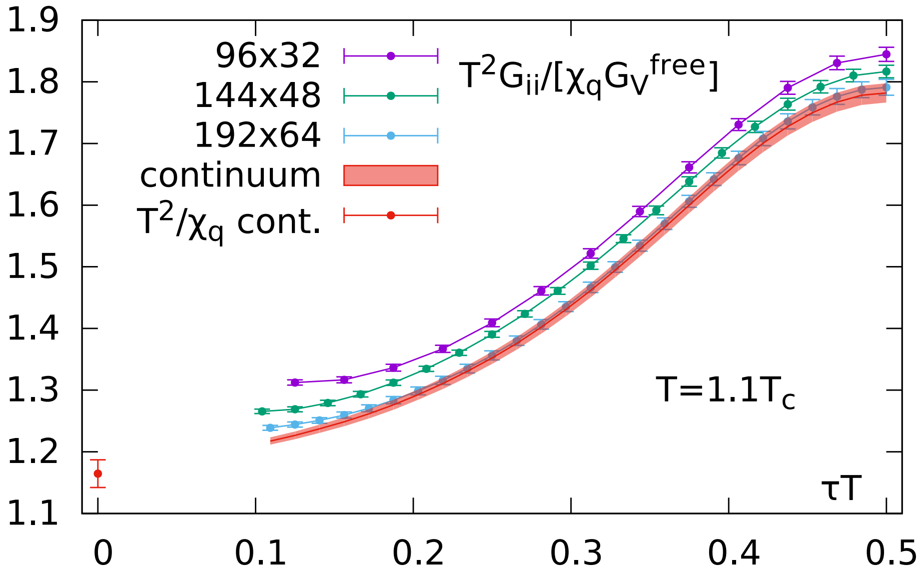
$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T) \qquad K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

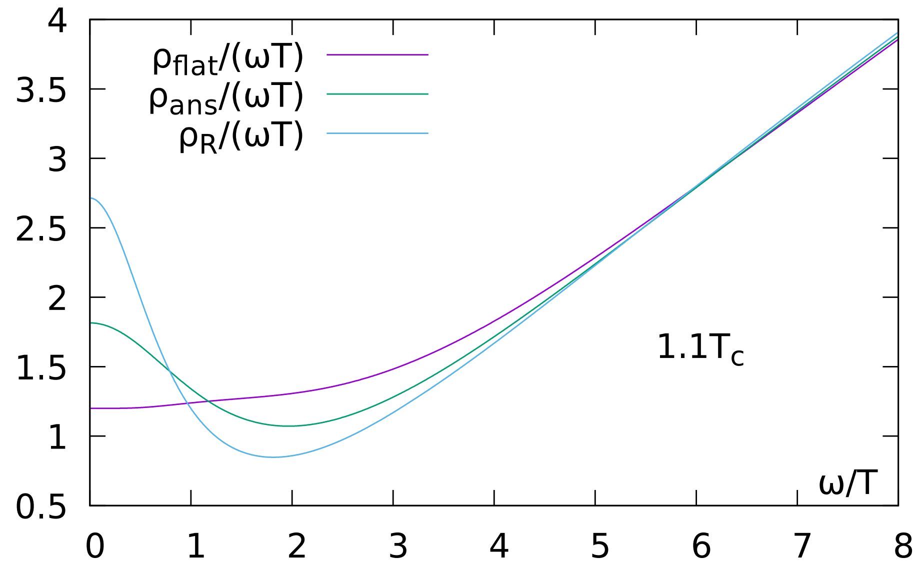
Ansatz for the (non-perturbative) transport contribution: $\rho_{BW}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2}$

and perturbative constraints for the UV part of the spectral function

$$\rho_R(\omega) = \rho_{BW}(\omega) + \frac{3\omega^2}{4\pi}[1-2n_{\mathrm{F}}(\frac{\omega}{2})]R(\omega^2)$$ (5-loop vacuum + LO thermal correction)

Fit to continuum extrapolated vector-meson correlation function $G_{ii}(\tau,T)$

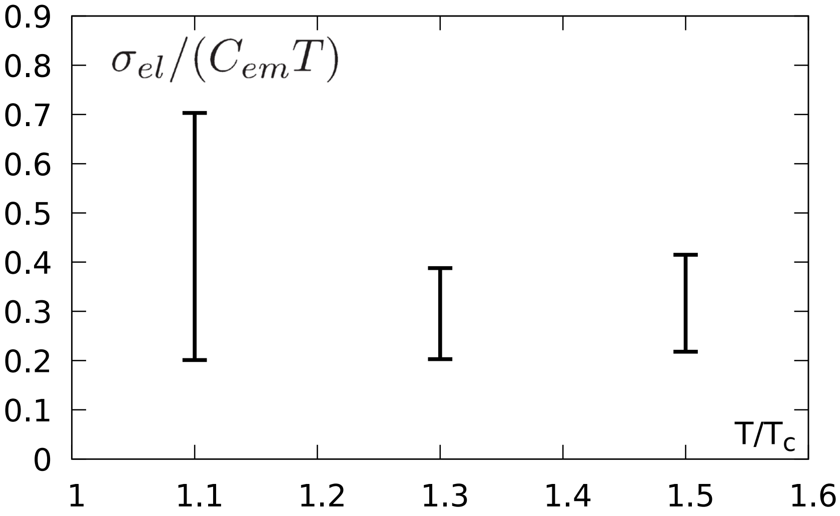




continuum estimate for the of the electrical conductivity

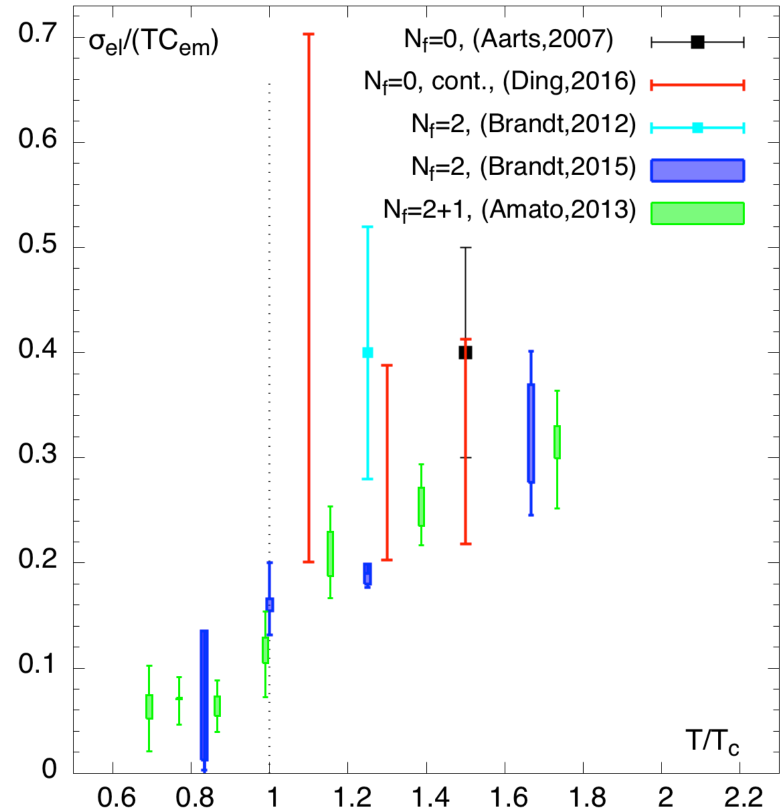
lower and upper limits from analysis of different classes of spectral functions:

$$\frac{\sigma_{el}}{C_{em}T} = \frac{1}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$



[H-T.Ding, F.Meyer, OK, PRD94(2016)034504]

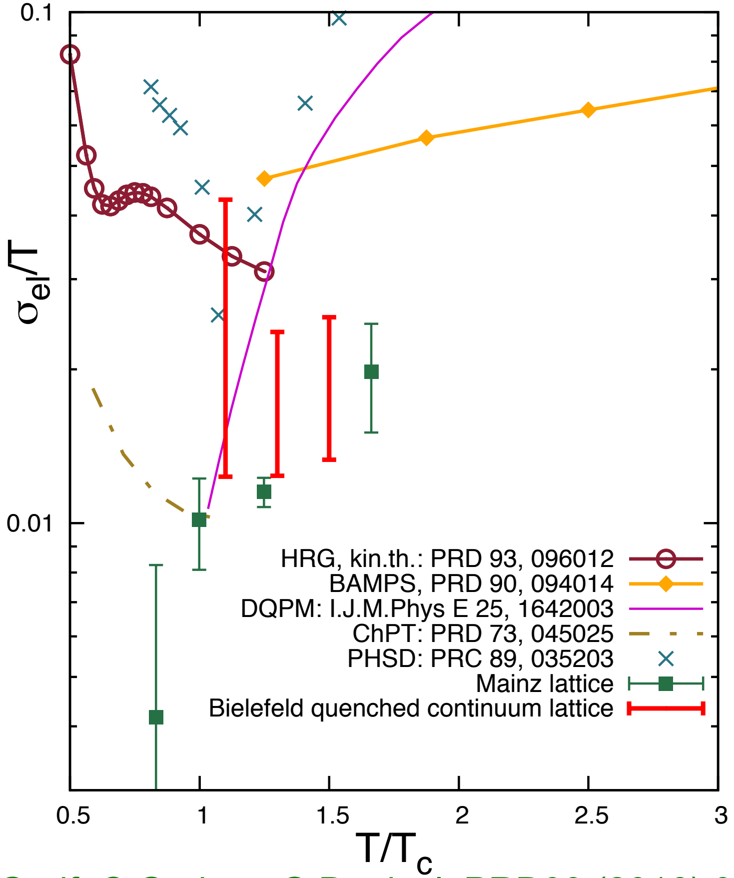
comparison of different lattice results
(Plot courtesy of A.Francis)



[G.Aarts et al., PRL 99 (2007) 022002,
H-T.Ding, F.Meyer, OK, PRD94(2016)034504,
B.B.Brandt et al., JHEP 1303 (2013) 100,
Brandt et al., PRD93 (2016) 054510,
A.Amato et al., PRL 111 (2013) 172001]

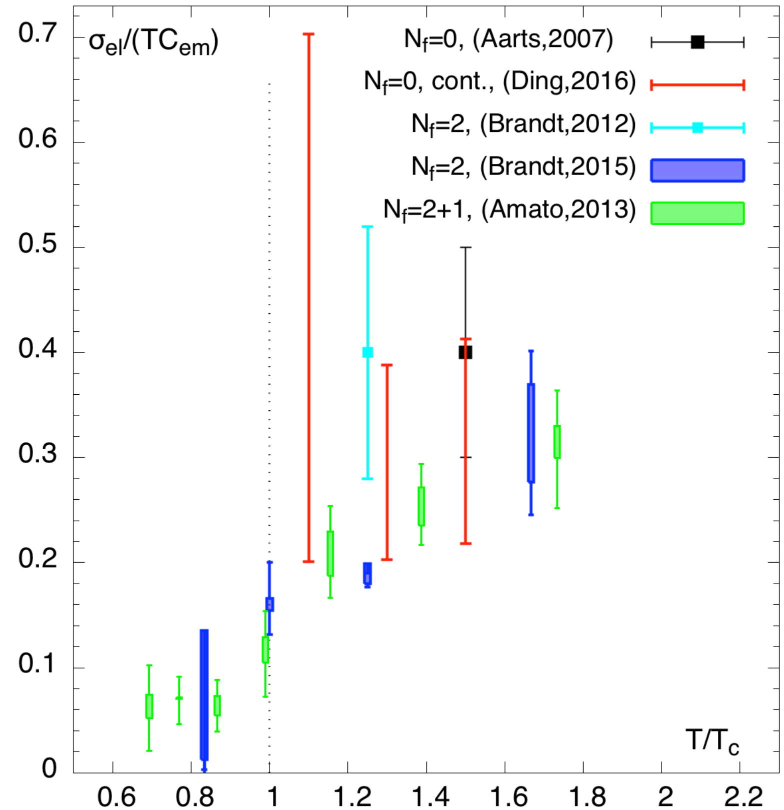
Electrical conductivity of the QGP

compared to calculations in partonic transport approaches



[M.Greif, C.Greiner, G.Denicol, PRD93 (2016) 096012]

comparison of different lattice results
(Plot courtesy of A.Francis)

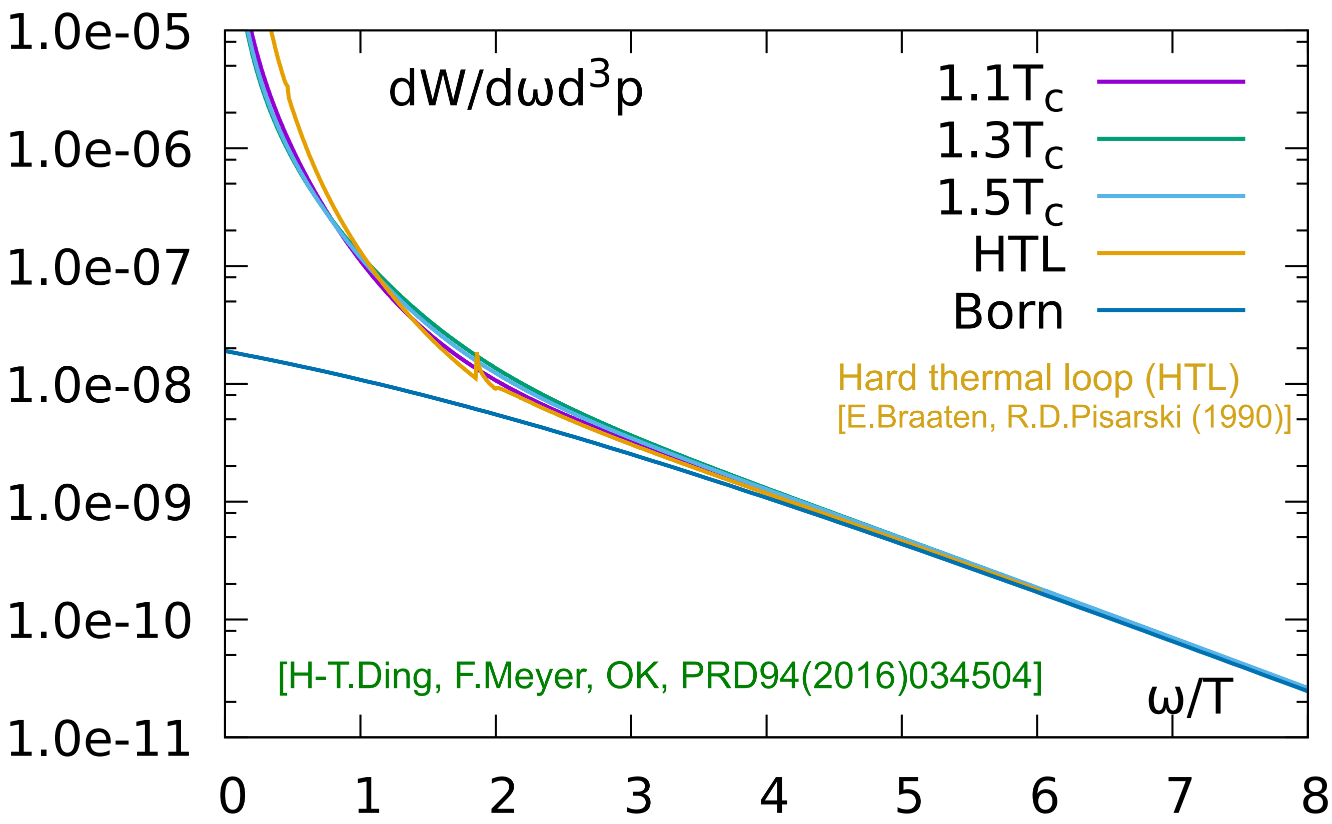


[G.Aarts et al., PRL 99 (2007) 022002,
H-T.Ding, F.Meyer, OK, PRD94(2016)034504,
B.B.Brandt et al., JHEP 1303 (2013) 100,
Brandt et al., PRD93 (2016) 054510,
A.Amato et al., PRL 111 (2013) 172001]

Progress in determining transport coefficients, although systematic uncertainties still need to be reduced in the future.

Dileptonrate directly related to vector spectral function:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\,\rho_{\mathbf{V}}(\omega,\mathbf{T})$$



Photonrate directly related to vector spectral function (at finite momentum):

$$\omega\frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x\mathrm{d}^3q} = \frac{5\alpha}{6\pi^2}\frac{1}{e^{\omega/T}-1}\rho_V(\omega=|\vec{k}|,\vec{k},T)$$

Perturbative knowledge on the vector spectral function

Non-interacting limit, “Born rate” for large invariant mass M>>πT, with $M^2=\omega^2+k^2$

$$\rho_{\mathrm{V}}(\omega,\mathbf{k}) \;=\; \frac{N_{\mathrm{c}} T M^2}{2\pi k}\left\{\ln\left[\frac{\cosh(\frac{\omega+k}{4T})}{\cosh(\frac{\omega-k}{4T})}\right] - \frac{\omega\,\theta(k-\omega)}{2T}\right\}\;,$$

[G. Aarts and J.M. Martinez Resco, NPB 726 (2005) 93]

Leading-log order for invariant mass M=0: [J.I. Kapusta et al.,PRD44 (1991) 2774, R. Baier et al. Z.Phys.C53 (1992) 433]

$$\rho_{\mathrm{V}}(k,\mathbf{k}) \;=\; \frac{\alpha_{\mathrm{s}} N_{\mathrm{c}} C_{\mathrm{F}} T^2}{4}\ln\left(\frac{1}{\alpha_{\mathrm{s}}}\right)\left[1-2n_{\mathrm{F}}(k)\right] + \mathcal{O}(\alpha_{\mathrm{s}} T^2)\;,$$

Complete leading order for invariant mass M=0:

[Arnold, Moore, Yaffe, JHEP11(2001)57 and JHEP12(2001)9]

NLO at M = 0: [J.Ghiglieri et al., JHEP 1305 (2013) 010]

NLO at M $\sim$ gT: [J.Ghiglieri and G.D.Moore, JHEP 1412 (2014) 029]

NLO at M $\sim$ πT: [M.Laine, JHEP 1311 (2013) 120]

N^4LO at M >> πT: [S. Caron-Huot, PRD79 (2009) 125009, P.A.Baikov et al. PRL101 (2008) 012002]

Hydrodynamic regime

Vector spectral function in the hydrodynamic regime for $\omega, k \lesssim \alpha_s^2 T$:

$$\frac{\rho_{\mathrm{V}}(\omega,\mathbf{k})}{\omega} = \left(\frac{\omega^2 - k^2}{\omega^2 + D^2 k^4} + 2\right)\chi_{\mathrm{q}} D$$

with the quark number susceptibility: $$\chi_{\mathrm{q}} \equiv \int_0^\beta \mathrm{d}\tau \int_{\mathbf{x}} \langle V^0(\tau,\mathbf{x}) V^0(0)\rangle$$

and the diffusion coefficient: $$D \equiv \frac{1}{3\chi_{\mathrm{q}}} \lim_{\omega\to 0^+} \sum_{i=1}^{3} \frac{\rho^{ii}(\omega,\mathbf{0})}{\omega}$$

which relate to the electric conductivity: $$\sigma = e^2 \sum_{f=1}^{N_{\mathrm{f}}} Q_f^2 \chi_{\mathrm{q}} D$$

In this limit the (soft) photon rate becomes: $$\frac{\mathrm{d}\Gamma_\gamma(\mathbf{k})}{\mathrm{d}^3\mathbf{k}} \overset{k \lesssim \alpha_{\mathrm{s}}^2 T}{\approx} \frac{2T\sigma}{(2\pi)^3 k}$$

In the AdS/CFT framework the vector spectral function has the same infrared structure

and here numerical result can make predictions beyond the hydro regime

[S.Caron-Huot, JHEP12 (2006) 015]

Although not QCD, it may give qualitative insight into the structures at small ω and k

pQCD spectral function used in our analysis

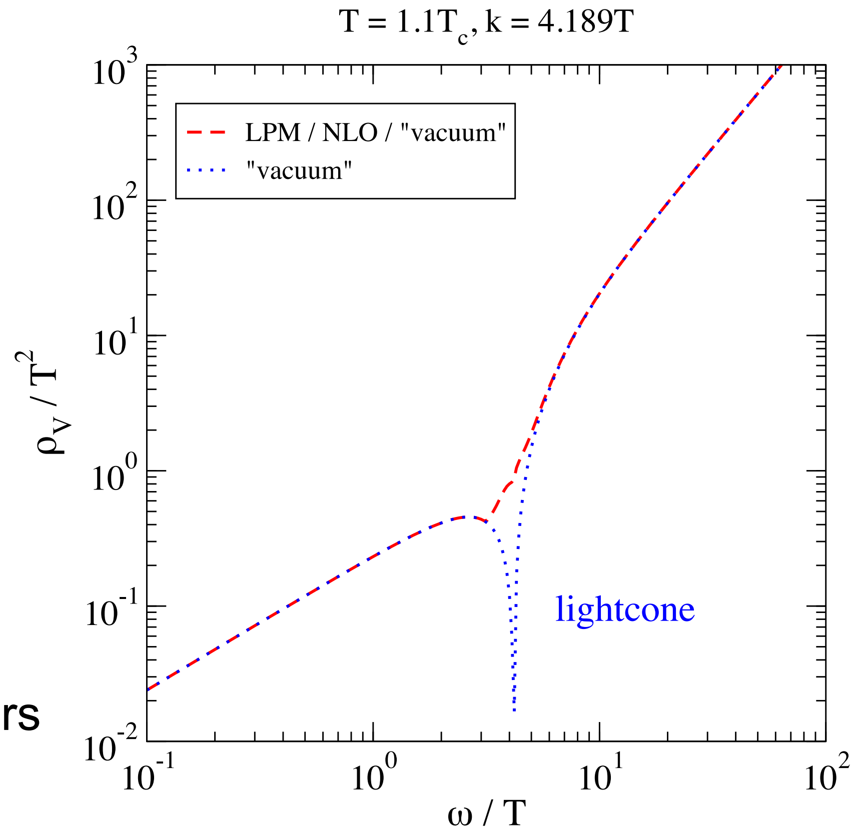
to allow for non-perturbative effects

and to analyze how far pQCD can be trusted

we model the infrared behavior

assuming smoothness at the light cone

and fit to continuum extrapolated lattice correlators



$3T < \omega < 10T$: [J.Ghiglieri and G.D.Moore, JHEP 1412 (2014) 029]

$\omega > 10T$: [I. Ghisoiu and M.Laine, JHEP 10 (2014) 84, arXiv:1407.7955]

$\omega >> 10T$: [M.Laine, JHEP 1311 (2013) 120]

interpolation between the different regimes: www.laine.itp.unibe.ch/dilepton-lattice

$(5+2\ n_{max})^{th}$ order polynomial Ansatz at small ω:

$$\rho_{\rm fit} \equiv \frac{\beta\,\omega^3}{2\omega_0^3}\left(5-\frac{3\omega^2}{\omega_0^2}\right)-\frac{\gamma\,\omega^3}{2\omega_0^2}\left(1-\frac{\omega^2}{\omega_0^2}\right)+\sum_{n=0}^{n_{\rm max}}\frac{\delta_n\omega^{1+2n}}{\omega_0^{1+2n}}\left(1-\frac{\omega^2}{\omega_0^2}\right)^2$$

with the constraints to match smoothly with pQCD at ω_o

$$\rho_{\rm V}(\omega_0,\mathbf{k}) \equiv \beta\,, \quad \rho'_{\rm V}(\omega_0,\mathbf{k}) \equiv \gamma\,,$$

and n_{max}+1 free parameters

starting with a linear behavior at $\omega \ll$ T

smoothly matched to the perturbative spectral function at $\omega_0 \simeq \sqrt{k^2+(\pi T)^2}$

In the following we will use n_{max} = 0 and n_{max} = 1 for the fits to the lattice data

and to estimate the systematic uncertainties

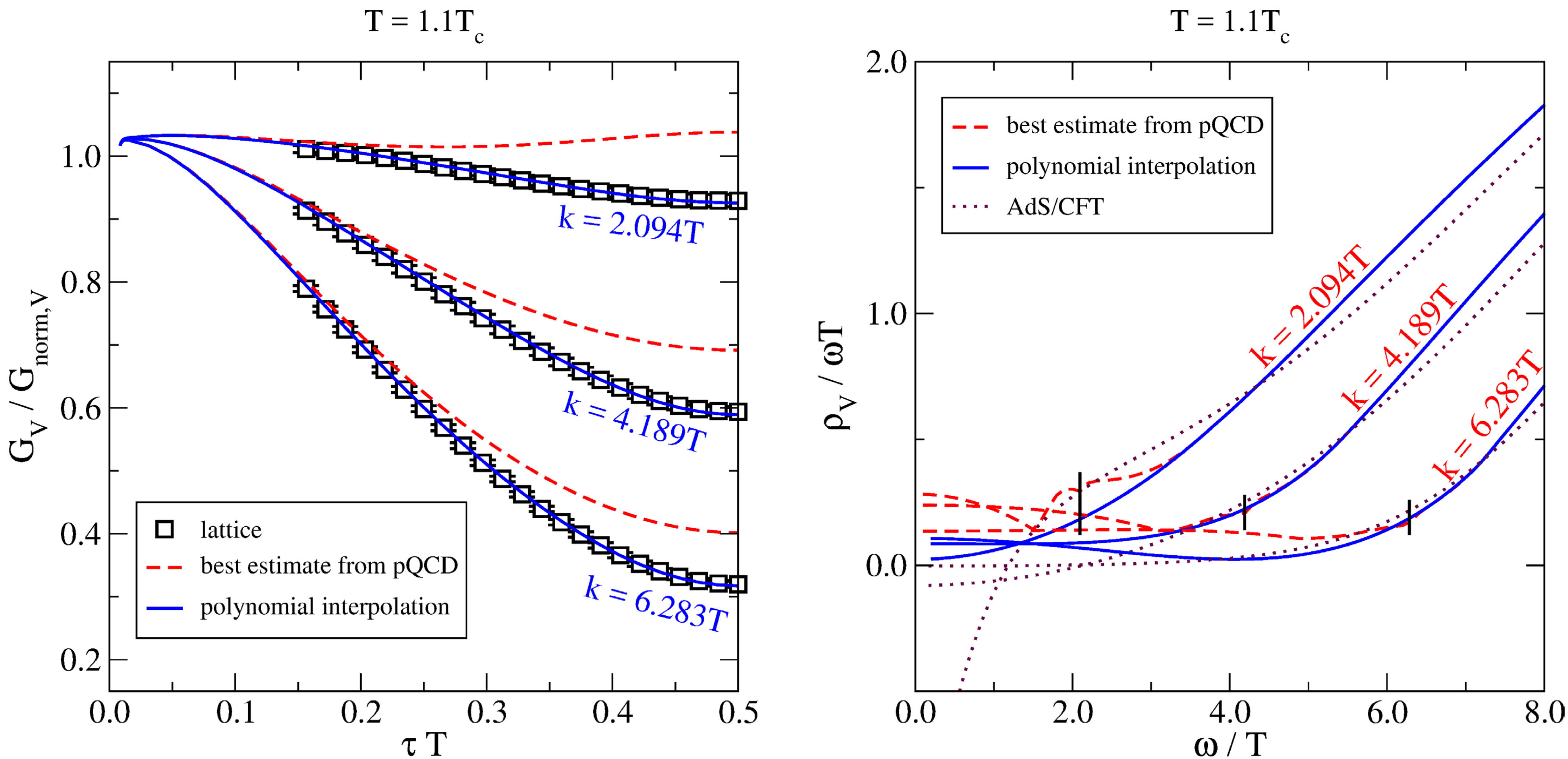
[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

Using vector correlation functions on large and fine lattices up to $196^3\times N_t$ with N_t=56,64

→ continuum extrapolation at finite momentum k

Using best perturbative knowledge to constrain the spectral function at large ω

→ fit a polynomial at small ω to extract the spectral function at the photon point ω = k



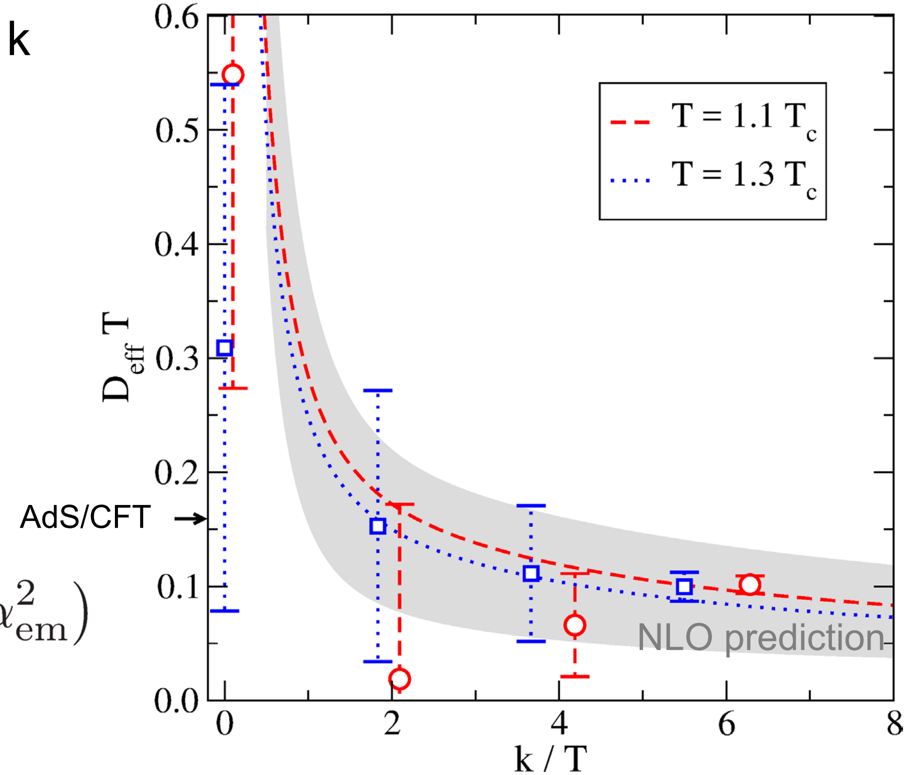
[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

The spectral function at the photon point ω = k

$$D_{\mathrm{eff}}(k) \equiv \begin{cases} \dfrac{\rho_{\mathrm{V}}(k,\mathbf{k})}{2\chi_{\mathrm{q}}k} & , \quad k>0 \\ \lim\limits_{\omega\to 0^+} \dfrac{\rho^{ii}(\omega,\mathbf{0})}{3\chi_{\mathrm{q}}\omega} & , \quad k=0 \end{cases} .$$

can be used to calculate the photon rate

$$\frac{d\Gamma_\gamma(\mathbf{k})}{d^3\mathbf{k}} = \frac{2\alpha_{\mathrm{em}}\chi_{\mathrm{q}}}{3\pi^2} n_{\mathrm{B}}(k) D_{\mathrm{eff}}(k) + \mathcal{O}\left(\alpha_{\mathrm{em}}^2\right)$$



becomes more perturbative at larger k, approaching the NLO prediction (valid for k>>gT)
[J. Ghiglieri, G.D. Moore, JHEP12 (2014) 029]

but non-perturbative for k/T < 3

Electrical conductivity obtained in the limit k→0 between the results from

AdS/CFT: $DT = \dfrac{1}{2\pi}$

[G.Policastro, D.T.Son,A.O.Starinets, JHEP09(2002)043]

LO perturbation theory [Arnold, Moore Yaffe, JHEP 05 (2003)]

using lattice value for χ_q/T^2: $DT = 2.9 - 3.1$

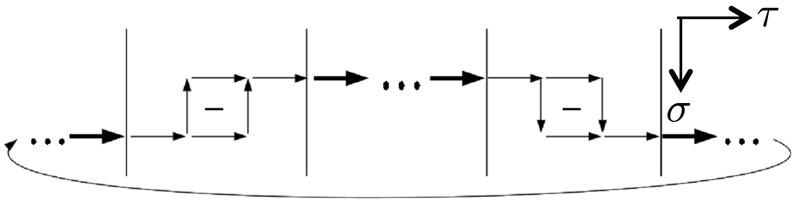
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) “color-electric correlator”

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012,
S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]

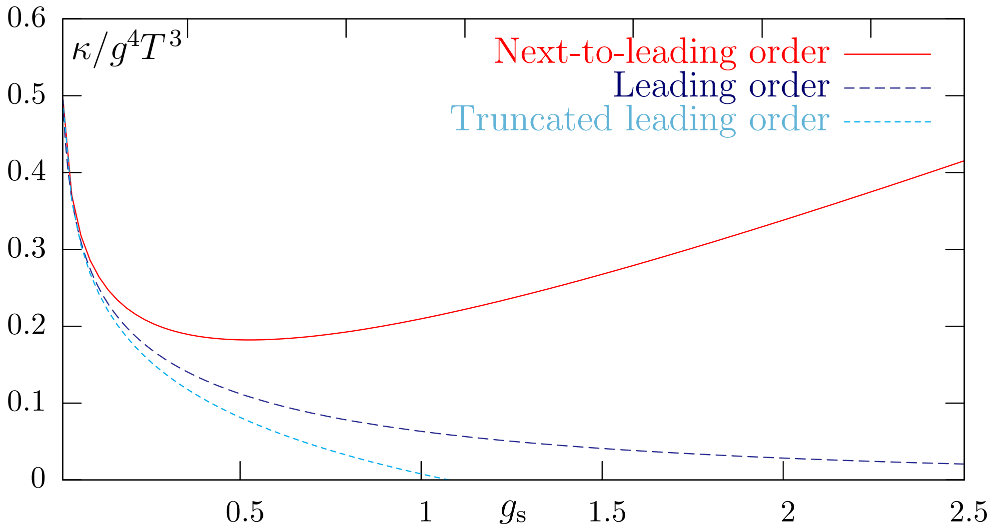
$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re}\,\mathrm{Tr}\left[U(\tfrac{1}{T};\tau)\, g\, E_i(\tau,\mathbf{0})\, U(\tau;0)\, g\, E_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re}\,\mathrm{Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$



$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$

NLO perturbative calculation:

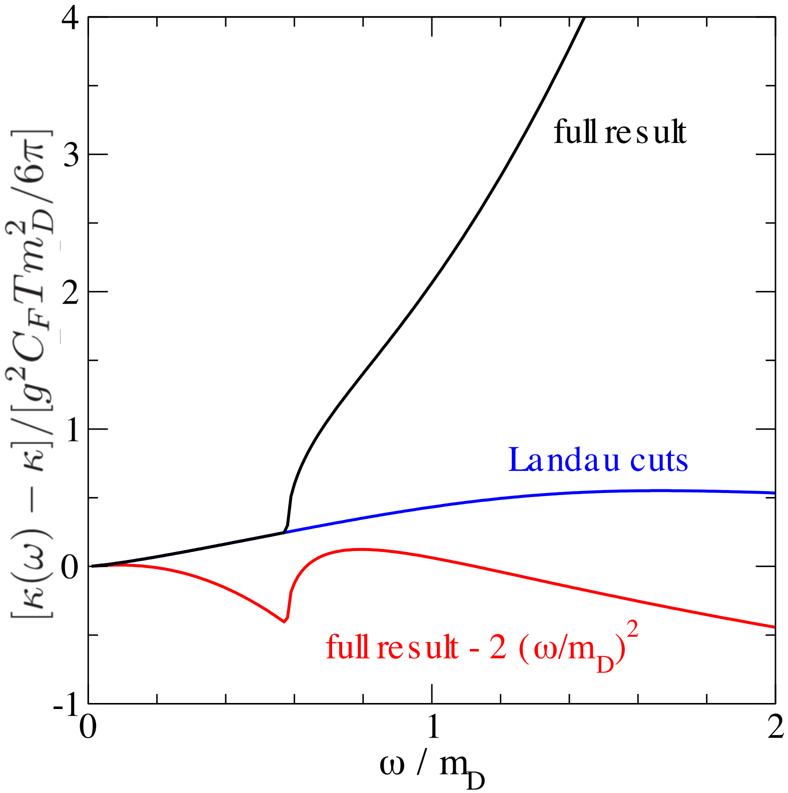
[Caron-Huot, G. Moore, JHEP 0802 (2008) 081]



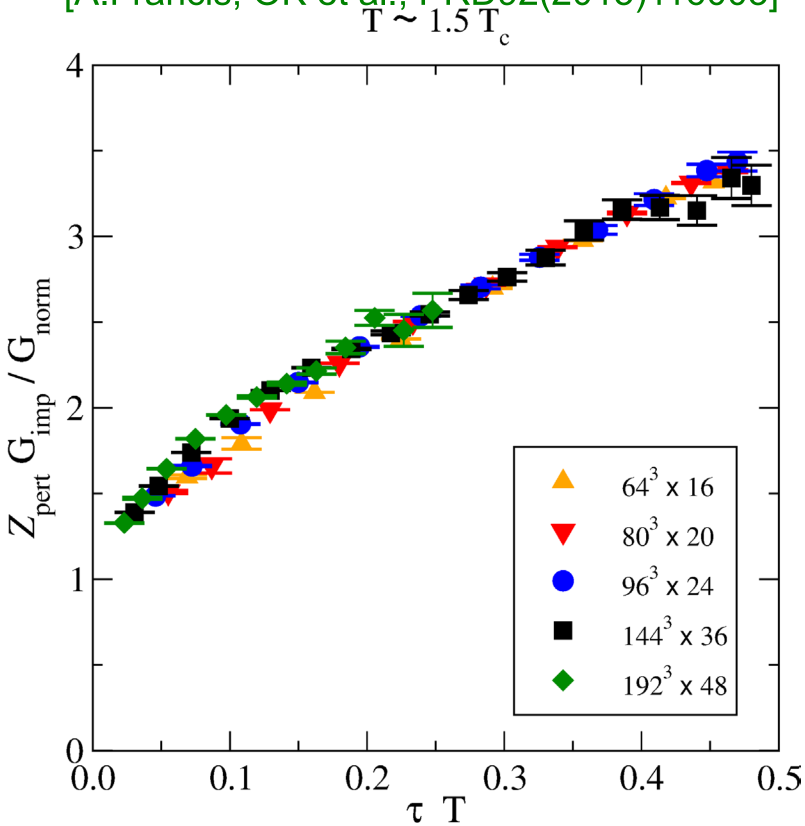
→ large correction towards strong interactions

→ non-perturbative lattice methods required

NLO spectral function in perturbation theory:
[Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]



Lattice QCD correlation functions:
[A.Francis, OK et al., PRD92(2015)116003]



in contrast to a narrow transport peak, from this a smooth limit is expected

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega}$$

qualitatively similar behavior also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]

no bound state contributions in this operator

$\omega \ll$T: linear behavior motivated at small frequencies

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

$\omega \gg$T: vacuum perturbative results and leading order thermal correction:

$$\rho_{\mathrm{UV}}(\omega) = \left[\rho_{\mathrm{UV}}(\omega)\right]_{T=0} + \mathcal{O}\left(\frac{g^4 T^4}{\omega}\right)$$

using a renormalization scale $\bar{\mu}_\omega = \omega$ for $\omega \gg \Lambda_{\overline{MS}}$ leading order becomes

$$\rho_{\mathrm{UV}}(\omega) = \Phi_{UV}(\omega)\left[1 + \mathcal{O}\left(\frac{1}{\ln(\omega/\Lambda_{MS})}\right)\right]$$

$$\Phi_{\mathrm{UV}}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi} \quad , \quad \bar{\mu}_\omega \equiv \max(\omega, \pi T)$$

here we used 4-loop running of the coupling

We use Ansätze that are consistent with these asymptotic behaviors

and model corrections to ρ_{IR} by a power series in ω

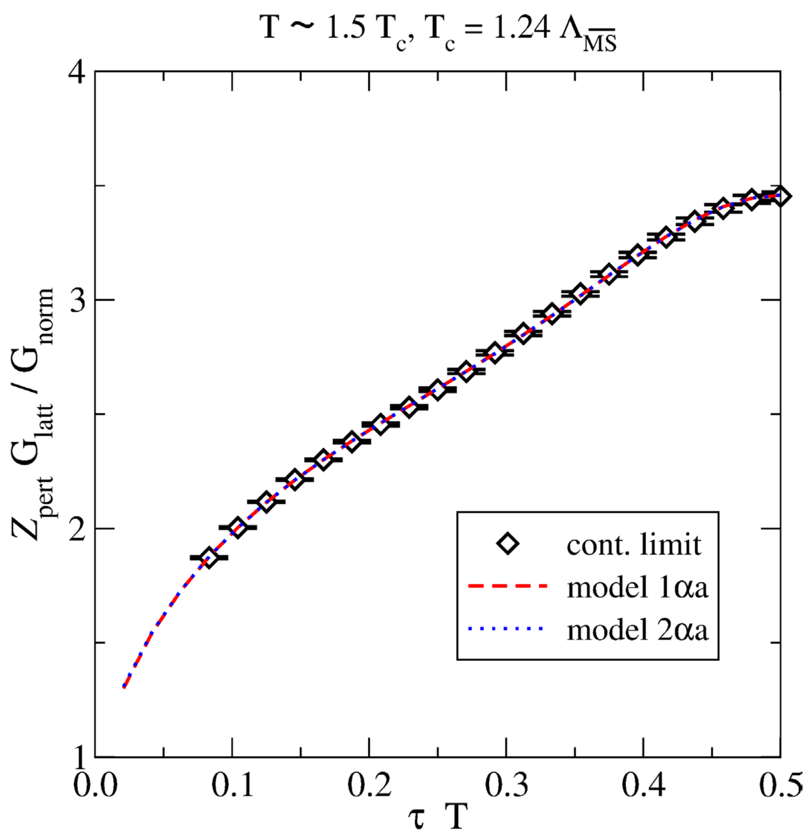
[A.Francis, OK et al., PRD92(2015)116003]

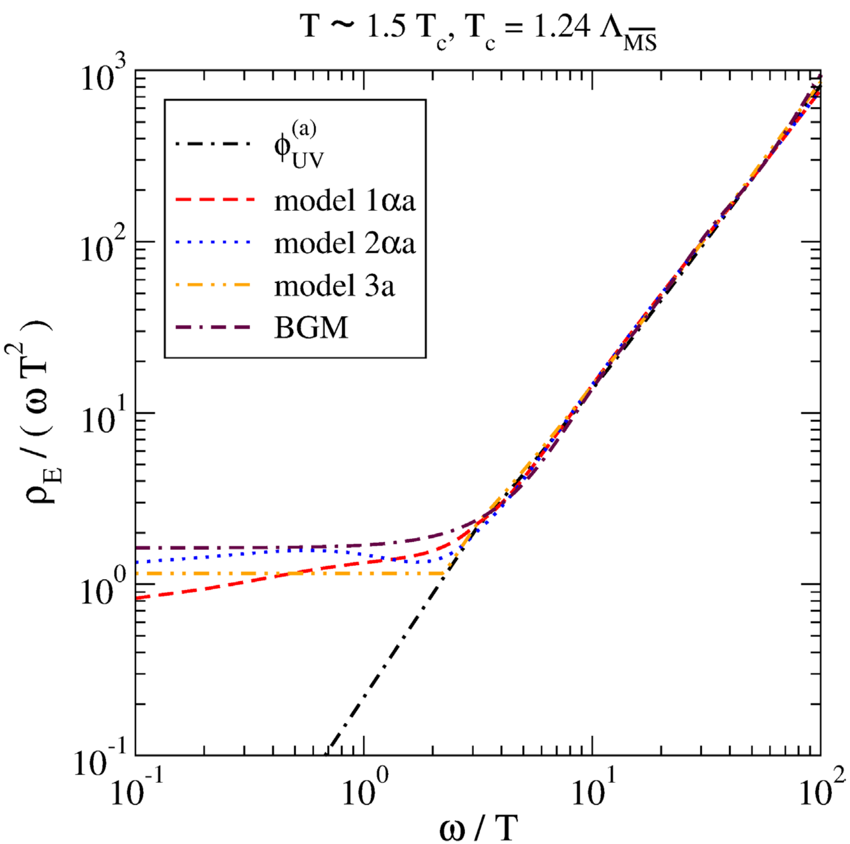
analysis of the systematic uncertainties by

using the best perturbative knowledge in the UV part of the spectral function

modeling corrections to ρ_{IR} by a power series in ω

and fitting to continuum extrapolated correlation function

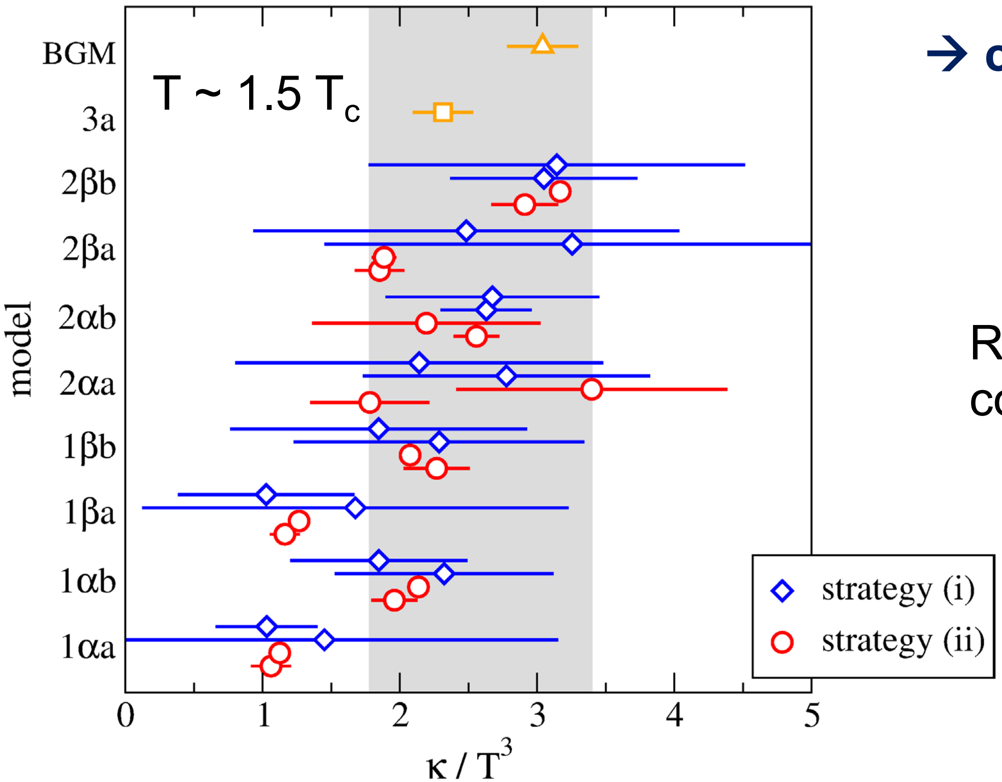




$$G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{model}}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}$$

[A.Francis, OK et al., PRD92(2015)116003]



Detailed analysis of systematic uncertainties

→ **continuum estimate of κ :**

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} = 1.8 \ldots 3.4$$

Related to diffusion coefficient D and drag coefficient η_D (in the non-relativistic limit)

$$2\pi T D = 4\pi \frac{T^3}{\kappa} = 3.7 \ldots 7.0$$

$$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

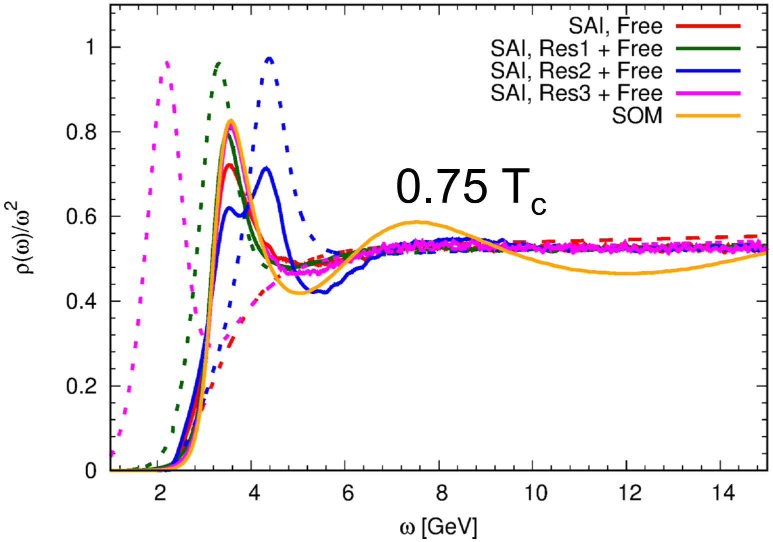
time scale associated with the kinetic equilibration of heavy quarks:

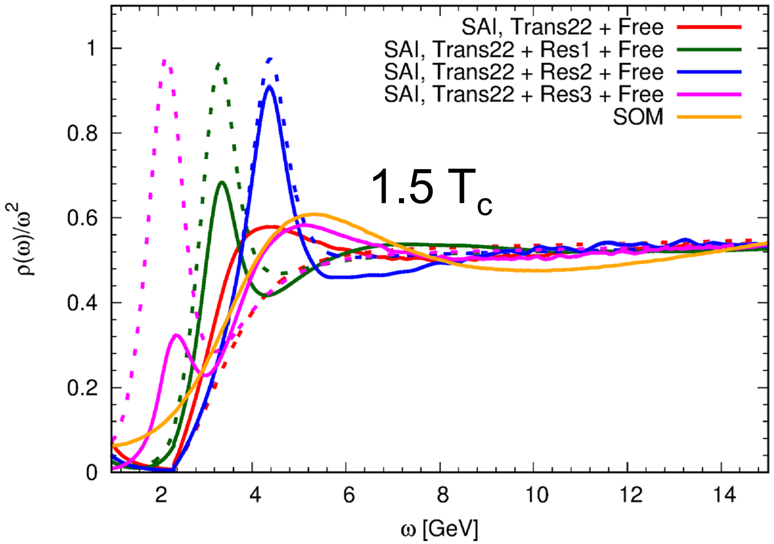
$$\tau_{\mathrm{kin}} = \frac{1}{\eta_D} = (1.8\ldots 3.4)\left(\frac{T_{\mathrm{c}}}{T}\right)^2 \left(\frac{M}{1.5\ \mathrm{GeV}}\right) \mathrm{fm/c}$$

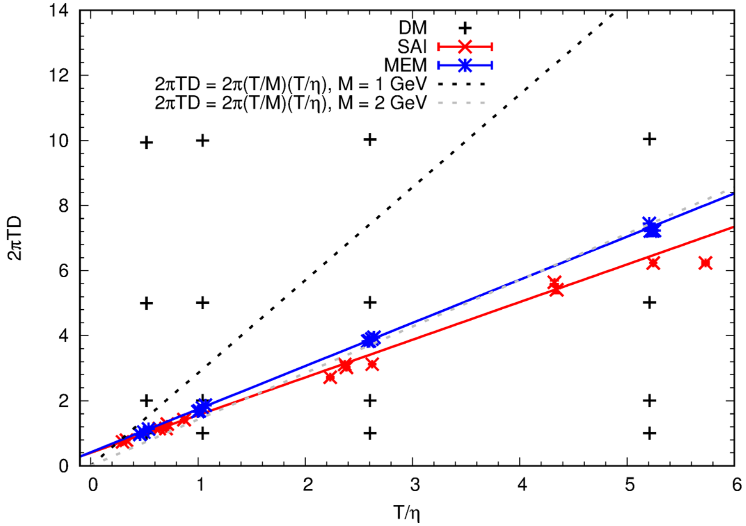
→ **close to T_c, $\tau_{kin} \simeq$ 1fm/c and therefore charm quark kinetic equilibration appears to be almost as fast as that of light partons.**

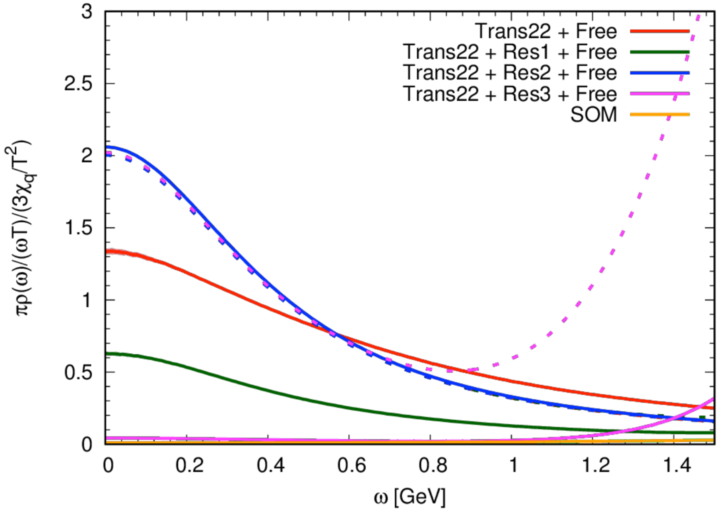
[H. Ohno, see Talk at QM 2017]

Using vector correlation functions on large and fine lattices up to $196^3 \times N_t$ with N_t=96,48 and Stochastic Analytical Interference method (SAI) based on Bayes' theorem:









From a careful analysis of systematic uncertainties: $2\pi TD = 1.6 - 7.0$

Conclusions

using **continuum extrapolated correlation functions** from Lattice QCD and

using phenomenologically inspired and **perturbatively constrained Ansätze**

allows to extracted transport properties and spectral properties

we obtained continuum estimates for

→ Electrical conductivity / Diffusion coefficients

→ Thermal dilepton rates

→ Thermal photon rates

next goals: continuum extrapolation for charm and bottom correlators

→ quark mass dependence of diffusion coefficient + sequential melting of quarkonia

The methodology developed in this studies within the quenched approximation

shall be extended to full QCD calculations for a realistic QGP medium

as close to T_c dynamical fermion degrees of freedom will become important